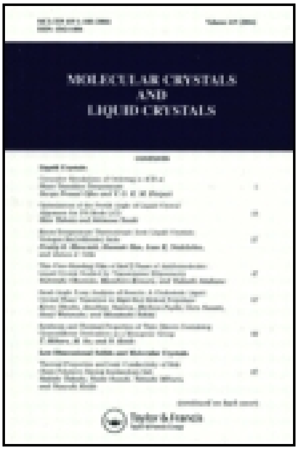


Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

TRANSIENT PATTERN DYNAMICS IN THE MAGNETIC NON-FRÉEDERICKSZ TWIST GEOMETRY

J. P. Casquilho [a] [b] , L. N. Gonçalves [a] [c] & J. L. Figueirinhas [c] [d]

[a] Departamento de Física, Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal

[b] CENIMAT, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal

[c] Centro de Física da Matéria Condensada, Universidade de Lisboa, Avenida Professor Gama Pinto 2, 1649-003 Lisboa Codex, Portugal

[d] Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa, Avenida Rovisco Pais, 1049-001 Lisboa Codex, Portugal

Version of record first published: 07 Jan 2010

To cite this article: J. P. Casquilho, L. N. Gonçalves & J. L. Figueirinhas (2004): TRANSIENT PATTERN DYNAMICS IN THE MAGNETIC NON-FRÉEDERICKSZ TWIST GEOMETRY, Molecular Crystals and Liquid Crystals, 413:1, 239-250

To link to this article: http://dx.doi.org/10.1080/15421400490437114

Mol. Cryst. Liq. Cryst., Vol. 413, pp. 239/[2375]–250/[2386], 2004

ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400490437114

TRANSIENT PATTERN DYNAMICS IN THE MAGNETIC NON-FRÉEDERICKSZ TWIST GEOMETRY

J. P. Casquilho
Departamento de Física
Universidade Nova de Lisboa, Quinta da Torre,
2829-516 Caparica, Portugal
and
CENIMAT, Faculdade de Ciências e Tecnologia,
Universidade Nova de Lisboa, Quinta da Torre,
2829-516 Caparica, Portugal

L. N. Gonçalves
Departamento de Física, Universidade Nova de Lisboa,
Quinta da Torre, 2829-516 Caparica, Portugal
and
Centro de Física da Matéria Condensada, Universidade de Lisboa,
Avenida Professor Gama Pinto 2, 1649-003 Lisboa Codex, Portugal

J. L. Figueirinhas
Centro de Física da Matéria Condensada, Universidade de Lisboa,
Avenida Professor Gama Pinto 2, 1649-003 Lisboa Codex, Portugal
and
Departamento de Física, Instituto Superior Técnico,
Universidade Técnica de Lisboa, Avenida Rovisco Pais,
1049-001 Lisboa Codex, Portugal

We present an analysis of the formation and evolution of the transient periodic pattern in the nematic director field reorientation in the magnetic non-Fréedericksz twist geometry and compare with experiments. The stability analysis of the magnetically induced director reorientation in a planar oriented nematic slab with rigid anchoring and with the viscoelastic parameters of 5CB shows that, when the magnetic field is not normal to the

Address correspondence to J. P. Casquilho, Departamento de Fisica and CENIMAT, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, Quinta da Torre, 2829-516 Caparica, Portugal.

undisturbed director $\boldsymbol{n}_0$, the transient hydrodynamic instability sets in for a periodic pattern that is oblique with respect to $\boldsymbol{n}_0$, in agreement with our experimental results for a nematic slab of 5CB. The analysis of the formation of the stripes show the equivalent of a tricritical point separating supercritical from subcritical bifurcations when the critical wavevector at the transition is plotted as a function of the reduced field. The analysis of the evolution of the stripes show that the amplitude of the periodic perturbations only grows significantly near the Fréedericksz geometry. This is also in agreement with our experimental results, where the stripes are only observable in the vicinity of this geometry. Our results show that there is a favourable range of the magnetic field intensity for the instability growth which corresponds to a minimum of the obliqueness of the stripes. We also show that the time for reaching the maximum distortion amplitude decreases with increasing magnetic field while for given field it increases when the Freedericksz limiting case is approached.

1. INTRODUCTION

The study of field induced transitions such as both the homogeneous [1,2] and inhomogeneous [3,4] Freedericksz transitions or in non-Freedericksz geometries [5,6] is an active subject in liquid crystal science. The formation of periodic stripes is, in general, determined by the competition between the elastic energy of the liquid crystal, the viscous dissipation, the interaction of the liquid crystal with the applied field, and the surface anchoring energy. It is the goal of this work to present an analysis of the formation and evolution of the transient periodic pattern in the nematic director field reorientation in the magnetic non-Fréedericksz twist geometry and compare with experiments. We study both theoretically and experimentally a sealed sample of 5CB between two parallel plates with planar boundary conditions and rigid anchoring.

The stability of the uniform director field reorientation with respect to periodic perturbations is studied as a function of the magnetic field $\mathbf{H}$, the angle α between $\mathbf{H}$ and the initial homogeneous nematic director $\mathbf{n}_0$ ($\mathbf{H}$ not normal to $\mathbf{n}_0$) and the nematic viscoelastic parameters. This perturbation method allows to linearize the dynamic equations around the uniform reorientation $u(t)$ taken at a time t after the magnetic field is applied [7]. Three cases are studied:

1. $u(t = 0) = 0$;
2. $u(t << \tau_0) << 1$, where τ_0 is the uniform reorientation time, which allows to linearize $u(t)$;
3. the complete (non-linear) solution for $u(t)$.

1. The zero order theory in $u(t)$ allows to explain the formation of spatial periodic director structures in the non-equilibrium nematic sample. The

case of a director reorientation with a pattern of periodic stripes normal to $\mathbf{n}_0$ allows to obtain analytical expressions for the critical wave vector and for the critical control parameters (H and α) [7], which shows that the periodic mode is cut off at a higher reduced field when the magnetic field acts away from the normal direction. In this case of normal stripes the bifurcation is supercritical. The numerical solution of the perturbation equations with the parameters of 5CB as reported in this work shows that a stripe pattern oblique to $\mathbf{n}_0$ is selected in this case. This is in agreement with the experimental results for a slab of 5CB reported here. These numerical results show the equivalent of a tricritical point separating supercritical from subcritical bifurcations when the critical wavevector at the transition is plotted as a function of the reduced field.

2. The first order theory in $u(t)$ was worked out for normal stripes which allowed to obtain analytical results for the evolution of the periodic pattern [6]. For $\alpha < \pi/2$ the results predict that the periodic modes selected in each instant have progressively smaller wave vectors as the director reorients back to equilibrium. It is also predicted that the initially growing amplitude of the periodic modes gets damped after a critical time and eventually vanishes. Consequently, it does not give way to inversion walls as in the $\alpha = \pi/2$ case but instead an uniform reorientation regime eventually develops. This is in qualitative agreement with the experimental observations reported here.

3. The non-linear theory in $u(t)$ is used in this work to obtain numerical results with the parameters of 5CB and for a 50 μm cell width. A periodic pattern that is oblique with respect to $\mathbf{n_0}$ shows up, in agreement with our experimental results for a nematic 50 μm slab of 5CB. While the results agree with the zero and the first order theories, they also show that the amplitude of the periodic perturbations only grows significantly near $\alpha = \pi/2$, predicting that the stripes should only be observable in the vicinity of the Freedericksz geometry. This is also in agreement with the experimental results. Our results show that there is a favourable range of the magnetic field intensity for the instability growth which corresponds to a minimum of the obliqueness of the stripes. We also show that the time for reaching the maximum distortion amplitude decreases with increasing magnetic field while for given field it increases when the Freedericksz limiting case is approached.

2. FORMATION OF THE PERIODIC PATTERN

We consider a nematic aligned monodomain between two parallel plates with planar boundary conditions and rigid anchoring and with positive anisotropy of the magnetic susceptibility χ_a. The magnetic field $\mathbf{H}$ is applied at an angle α with respect to the initial homogeneous director $\mathbf{n}_0$. We study the stability of the uniform reorientation in respect to a (spatially) periodic reorientation where the director remains in the sample plane such that the distortion angle θ depends on the height z and the position x along the axis defined by $\mathbf{n}_0$ and the position y. This corresponds to a twist-splay-bend deformation of the director and to a pattern of periodic stripes oblique to $\mathbf{n}_0$ as observed with 5CB (see next section). Accordingly, the following magnetic, velocity and director fields are considered:

$$\begin{aligned} H_x &= H\cos\alpha, H_y = H\sin\alpha, H_z = 0 \\ &v_x(x,y,z,t),\ v_y(x,y,z,t),\ v_z = 0 \\ n_x &= \cos\theta(x,y,z,t), n_y = \sin\theta(x,y,z,t), n_z = 0 \end{aligned} \tag{1}$$

We now follow the perturbation method described in [7]: we write the Ericksen-Leslie equations [8] for the fields (1) and take the following functions for the velocity and the director fields:

$$\begin{aligned} v_x(x,y,z,t) &= 0 + \xi_{v_x}(x,y,z,t) \\ v_y(x,y,z,t) &= 0 + \xi_{v_y}(x,y,z,t) \\ \theta(x,y,z,t) &= u(t) + \xi_\theta(x,y,z,t) \end{aligned} \tag{2}$$

In the rhs of (2) the first terms correspond to the uniform reorientation and the second terms correspond to the perturbations of the velocity and the director fields respectively. To first order, the perturbation equations around $u(t=0)=0$ are:

$$\begin{aligned} \rho\frac{\partial}{\partial t}\left(v_{x,y} - v_{y,x}\right) = {} & 2v_1 v_{x,xxy} + \eta_b v_{x,yyy} + \eta_4 v_{y,xyy} + \eta_b v_{x,zzy} - \eta_6 v_{x,yxx} \\ & - \eta_c v_{y,xxx} - \alpha_4 v_{y,yyx} - \eta_a v_{y,zzx} + \alpha_3\frac{\partial}{\partial t}\xi_{\theta,yy} \\ & - \alpha_2\frac{\partial}{\partial t}\xi_{\theta,xx} - \gamma_2\left.\frac{du(t)}{dt}\right|_{t=0}\left(\xi_{\theta,xy} + \xi_{\theta,yx}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} \gamma_1\frac{\partial}{\partial t}\xi_\theta + \alpha_3 v_{x,y} + \alpha_2 v_{y,x} + \chi_a H^2\cos 2\alpha \\ - K_1\xi_{\theta,yy} - K_2\xi_{\theta,zz} - K_3\xi_{\theta,xx} = 0 \end{aligned} \tag{4}$$

where the parameters are the Leslie coefficients α_i, $i = l,\ldots,5$, $\alpha_6 = \alpha_3 + \alpha_2 + \alpha_5$, the rotational viscosities $\gamma_1 = \alpha_3 - \alpha_2$ and $\gamma_2 = \alpha_3 + \alpha_2$, the Miesowicz viscosities $2\eta_a = \alpha_4$, $2\eta_b = \alpha_3 + \alpha_4 + \alpha_6$ and $2\eta_c = \alpha_4 + \alpha_5 - \alpha_2$, the elongational viscosity $2v_1 = \alpha_1 + \alpha_4 + \alpha_5 + \alpha_6$ and the viscosities $2\eta_4 = \alpha_4 + \alpha_6 - \alpha_3$ and $2\eta_6 = \alpha_4 + \alpha_5 + \alpha_2$, and the Frank elastic constants K_{i}, i = 1,2,3 [8]. In (3) we take [6]:

$$\left.\frac{du(t)}{dt}\right|_{t=0} = \frac{1}{2}\frac{\chi_a H^2}{\gamma_1}\sin(2\alpha) \tag{5}$$

It is the term in $(du(t)/dt)_{t=0}$ in equation (3) that will allow to obtain an oblique stripe pattern for $\alpha \neq 90°$. We take the following *ansatze* for the perturbations:

$$\begin{aligned}\xi_{v_x}(x,y,z,t) &= -v_0(t)q_y \sin(q_x x + q_y y)\cos q_z z \\ \xi_{v_y}(x,y,z,t) &= v_0(t)q_x \sin(q_x x + q_y y)\cos q_z z \\ \xi_\theta(x,y,z,t) &= \theta_0(t)\cos(q_x x + q_y y)\cos q_z z\end{aligned} \tag{6}$$

where q_x, q_y and q_z are the cartesian components of the wavevector of the distortion, with $q_z = \pi/d$ where d is the sample thickness in the OZ direction. The functions (6) represent harmonic distortions that obey the boundary conditions at $z = \pm d/2$ corresponding to the no slip condition for the velocity and to planar alignment for the director [9]. This description of the velocity obeys the incompressibility condition.

In the following analysis we will simplify the problem neglecting the inertial term in the velocity equation (3). We checked for d up to 1 mm that the stability analysis of both the system (3,4) with (5,6) and the symplified system gives the same results. It is convenient to write the resulting equation in the following adimensional form:

$$\gamma_{ef}\frac{d\theta_0}{dt'} = \left(-a + \frac{b}{c}\frac{\gamma_2}{\gamma_1}q'_x q'_y h^2 \sin 2\alpha\right)\theta_0 \tag{7}$$

where $h = H/H_c$ is the reduced field with $H_c = (K_2\pi^2/\chi_a d^2)^{1/2}$ the critical field for the homogeneous twist Freederickzs transition [8], $t' = t/(\gamma_1/\chi_a H_c^2)$ is a reduced time, $\gamma_{ef} = 1 - b^2/\gamma_1 c$ is a reduced effective viscosity, $q'_x = q_x/q_z$ and $q'_y = q_y/q_z$ are the reduced components of the wave vector of the periodic mode and

$$a = 1 + \frac{K_1}{K_2}q'^2_y + \frac{K_3}{K_2}q'^2_x + h^2\cos 2\alpha \tag{8}$$

$$b = \alpha_2 q'^2_x - \alpha_3 q'^2_y \tag{9}$$

$$c = N q'^2_x q'^2_y + \eta_b q'^4_y + \eta_c q'^4_x + \eta_a q'^2_x + \eta_b q'^2_y, \quad N = \alpha_1 + n_b + \eta_c \tag{10}$$

The substitution of the ansatz $\theta_0(t') = \theta_0(0)\exp(\lambda t')$ in equation (7) yields the growth rate $\lambda(q'_x, q'_y, h, \alpha)$ of the periodic modes. Next we maximize this function in order to q'_x and q'_y, and we get the components q'_{xc} and q'_{yc} of the (reduced) critical wavevector and the critical growth rate $\lambda_c \equiv \lambda(q'_{xc}, q'_{yc})$. We will take q'_{xc} as the order parameter of our system. These components

TABLE 1 Parameters of 5CB [10]

$\alpha_1 = -0.066\,\mathrm{g\,cm^{-1}s^{-1}}$
$\alpha_2 = -0.77\,\mathrm{g\,cm^{-1}s^{-1}}$
$\alpha_3 = -0.042\,\mathrm{g\,cm^{-1}\ s^{-1}}$
$\alpha_4 = 0.634\,\mathrm{g\,cm^{-1}s^{-1}}$
$\alpha_5 = 0.624\,\mathrm{g\,cm^{-1}\ s^{-1}}$
$K_1 = 5.95 \times 10^{-7}$ dyne
$K_2 = 3.77 - 10^{-7}$ dyne
$K_3 = 7.86 - 10^{-7}$ dyne

have been calculated numerically with the viscoelastic parameters of 5CB of Table 1. In the Figure 1 the ratio q_{yc}/q_{xc} is plotted as a function of the angle $\alpha \leq 90^\circ$ for two different values of the reduced field h, showing that this ratio is smaller than one, increases from zero when α departs from 90° and gets smaller for larger reduced fields. The calculations for $\alpha > 90^\circ$ predict an inversion of the obliqueness of the stripes after $\alpha = 90^\circ$, which is in (qualitative) agreement with our experimental results (see next section).

Figure 2 shows the values of q'_{xc} and q'_{yc} at the transition homogeneous – periodic reorientation and the corresponding values of the critical angle α_c, as a function of the reduced field. For each value of h a scan in α was made

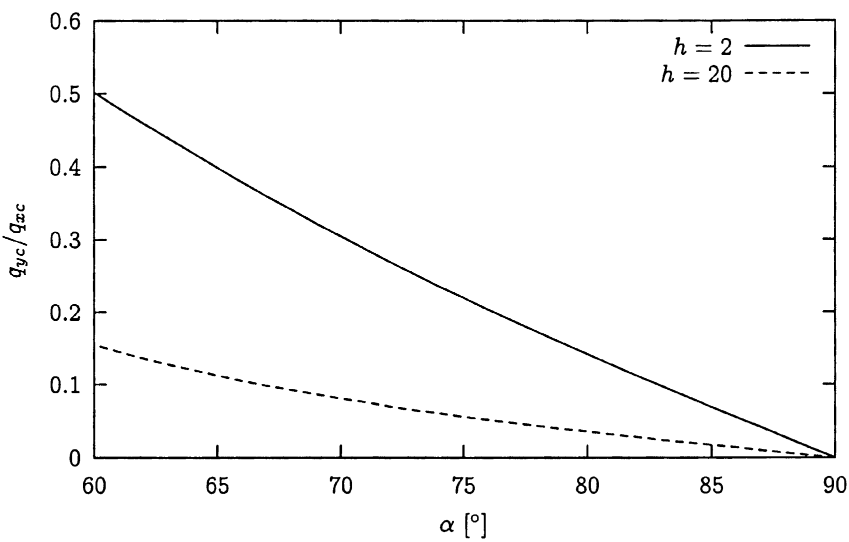


FIGURE 1 The ratio q_{yc}/q_{xc} is plotted as a function of the angle α. (1) $h = 2$; (2) $h = 20$.

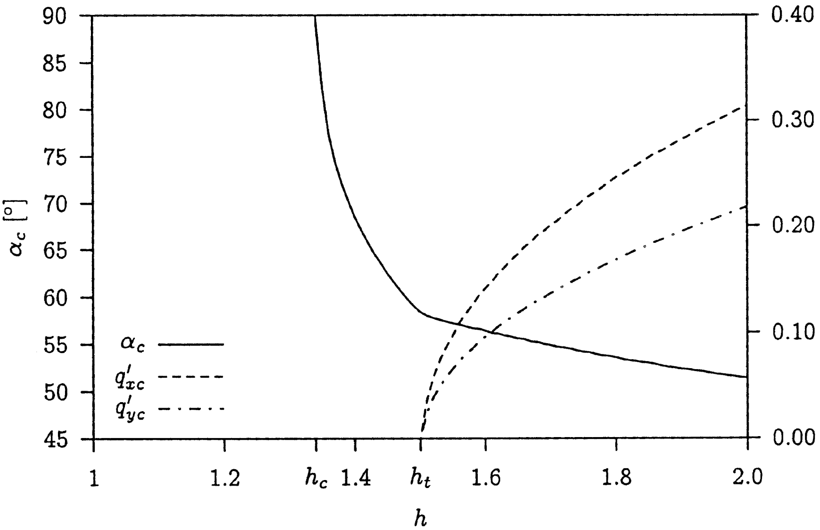


FIGURE 2 q'_{xc}, q'_{yc} and α_c at the transition aperiodic – periodic reorientation, as a function of the reduced field.

from 90° downwards. These results show at $h_c = 1.34$ a critical field for the growth of the periodic mode and at $h_t = 1.5$ the equivalent of a tricritical point separating continuous from discontinuous transitions.

3. EVOLUTION OF THE PERIODIC PATTERN

The time evolution of the periodic perturbation was investigated for $u(t,z) \neq 0$ where $u(t,z)$ is the (non-linear) homogeneous reorientation considering boundary conditions. The director and velocity equations were linearized in the perturbations ξ given by (6) around $u(t)$ and these equations were numerically integrated considering for ξ_θ as initial conditions a thermal distribution of modes. Details will be given elsewhere [11]. The determination of each mode amplitude at time t $\xi_\theta(t, \mathbf{q})$ was achieved by numeric integration of the linearized equations from $t = 0$ to t for specific values of q_x and q_y. The leading mode amplitude $\xi_\theta(t)$ and its wave vector $\mathbf{q}$ were found maximizing $\xi_\theta(t, \mathbf{q})$ as a function of $\mathbf{q}$. The time evolution of the leading mode amplitude $\xi_\theta(t)$ and wave vector $\mathbf{q}$ were determined for several magnetic field strengths and inclination angles α. It is seen that $\xi_\theta(t)$ reaches a maximum for a specific time t_m. In (Figures 3a–d) the values of $\xi_\theta(t_m)$, $q(t_m)$, t_m and the stripes tilt angle $\Psi = \mathrm{tg}^{-1}(q_\mathrm{y}/q_\mathrm{x})$ are plotted as a function of the field strength for $\alpha = 86°$. These results show that there is a favourable range of the magnetic field intensity for the instability growth which corresponds to a minimum of the obliqueness of the stripes, and that the time for reaching the maximum distortion amplitude decreases with increasing magnetic field while for given field it increases when the Freedericksz limiting case is approached.

The formation and evolution of the periodic stripes where also studied experimentally. A planar cell filled with 5CB was placed in the pole gap

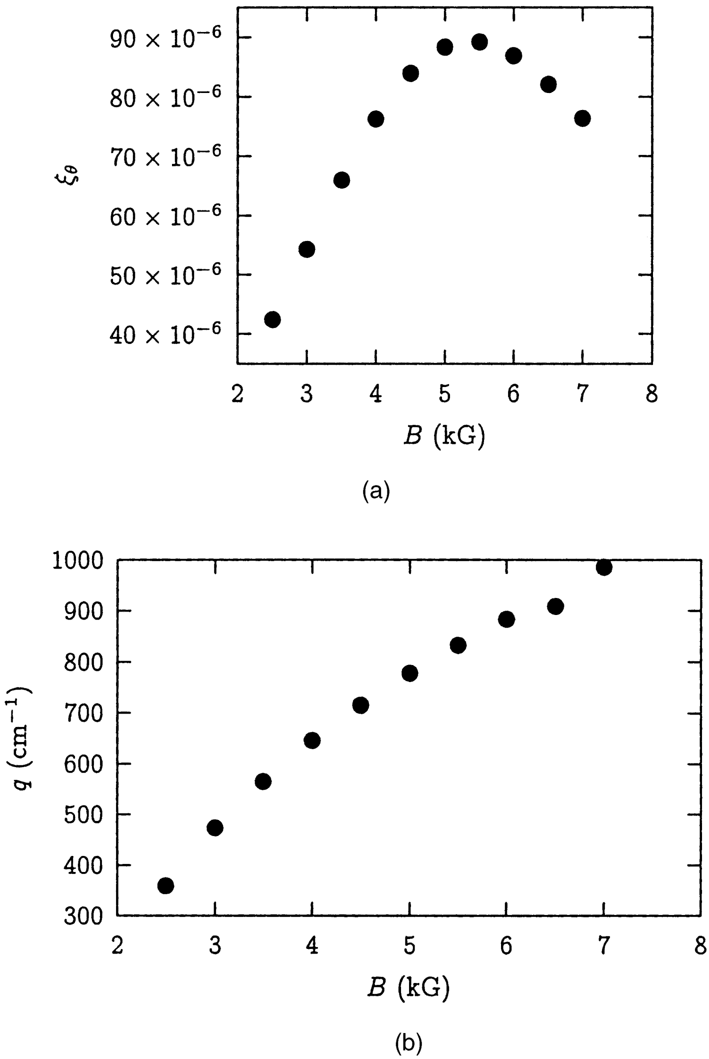
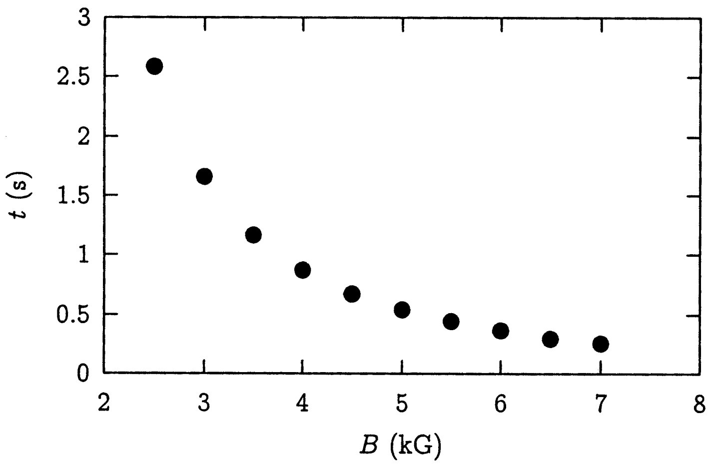
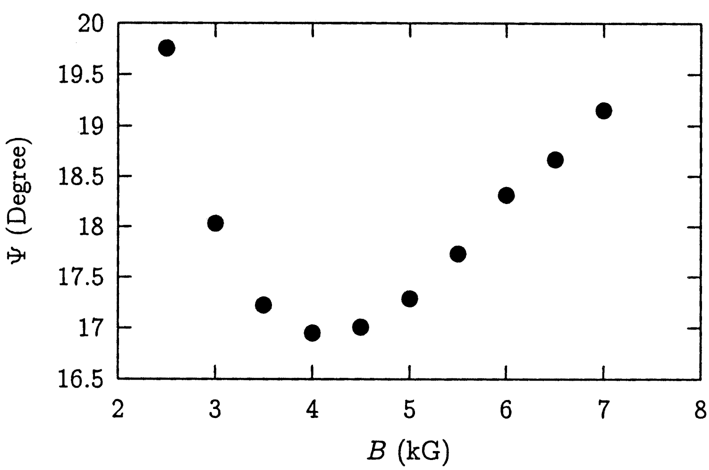


FIGURE 3 (a)$\xi_\theta(t_m)$ (b)$q(t_m)$ (c)t_m (d)$\Psi(t_m)$ v.s. B for $\alpha = 86°$.

of an electromagnet. Reflection patterns of sample cells at 10 degree incidence were observed directly in the magnetic field. A standard lamp fitted with a linear polarizer was used as the light source. The light reflected from the cell was recorded by means of a color video camera (320×240 pixels). Video output of the camera was captured into a PC. Pictures were taken at a rate of 25 frames/s. The sample cell was mounted on top of a temperature

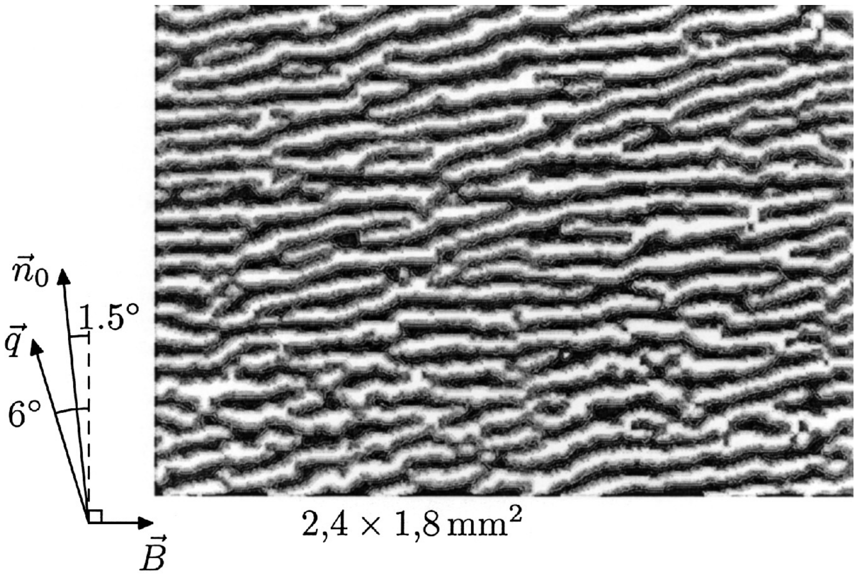


(c)

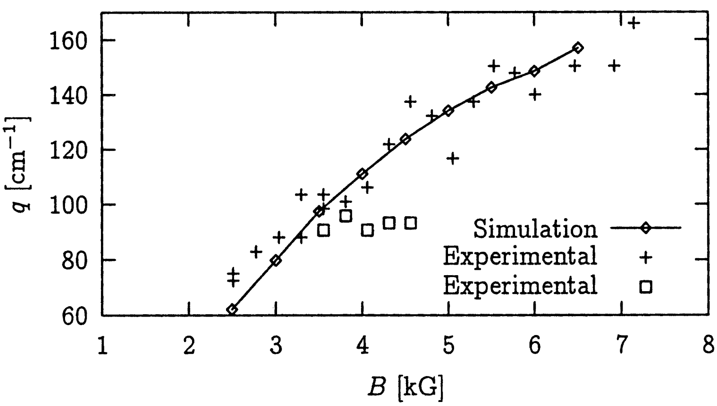


(d)

FIGURE 3 (Continued).



(a)



(b)

FIGURE 4 (a) Picture of the stripe pattern for $B = 5\,\text{kG}$ and $\alpha = 88.5°$. (b) Wave vector q v.s. applied magnetic field B for $\alpha = 88.5°$. (c) Stripes tilt angle Ψ v.s. rotation angle α for $B = 5\,\text{kG}$.

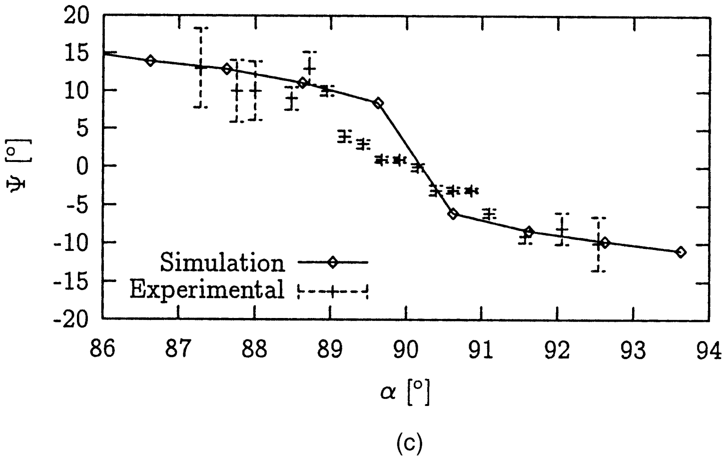


FIGURE 4 (Continued).

controlled sample holder which could be rotated by a mechanical system in steps of 0.25 degree around an axis normal to the cell plane. The sample cells have a thickness of 50 micron and were manufactured by E.H.C.co. Details of the experimental set-up and procedure will be given elsewhere [12]. The experimental results for q and Ψ show good agreement with the dependencies numerically found as seen in Figure 4. The picture shown in the Figure 4a is a single frame while the experimental points shown in the Figure 4c are obtained from an average of several frames. In the Figure 4b the lower experimental points correspond to the non-linear mode [12].

REFERENCES

[1] Demeter, G. & Kramer, L. (2001). *Phys. Rev. E. 64*, 020701.
[2] Yang Guochen & Zhang Suhua. (2002). *Liq. Cryst. 27*, n°5, 641.
[3] Krzyzanski, D. & Derfel, G. (2001). *Phys. Rev. E. 63*, 021702.
[4] Chevallard, C. & Clerc, M. C. (2001). *Phys. Rev. E. 65*, 011708.
[5] Polimeno, A., Orian, L., Martins, A. F., & Gomes, A. E. (2000). *Phys. Rev. E. 62*, 2288; Martins, A. F., Gomes, A. E., Polimeno, A., & Orian, L. (2000). *Phys. Rev. E. 62*, 2301.
[6] Casquilho, J. P. & Figueirinhas, J. L. (2002). *Liq. Cryst. 29* n°1, 127.
[7] Casquilho, J. P. (1999). *Liq. Cryst. 26*, n°4, 517.
[8] de Gennes, P. G. & Prost, J. (1993). *The Physics of Liquid Crystals*, 2nd edition (Oxford Clarendon Press).
[9] The stability analysis gives equivalent results with the *ansatze*:
$\xi_{v_x}(x,y,z,t) = \pm v_0(t) q_y \cos(q_x x + q_y y)\cos q_z z$
$\xi_{v_y}(x,y,z,t) = \mp v_0(t) q_x \cos(q_x x + q_y y)\cos q_z z$
$\xi_{\theta}(x,y,z,t) = \pm \theta_0(t) \sin(q_x x + q_y y)\cos q_z z$

[10] Ahlers, G. (1995). In: *Pattern Formation in Liquid Crystals*, chap. 5, Buka, A. & Kramer, L (Ed.), Springer.
[11] Figueirinhas, J. L. & Casquilho, J. P. To be published.
[12] Gonçalves, L. N., Casquilho, J. P., & Figueirinhas, J. L. (2002). ILCC; Gonçalves, L. N., Casquilho, J. P., Ribeiro, A. C., & Figueirinhas, J. L. To be published.